Solutions to Supplementary Problems

Chapter 2

Solution 2.1

a) First, the specific volume change/moisture content graph (f against m) has to be drawn, using:

$$(2.8):\ f = 100 \times \frac{V - V_0}{V_0} = 100 \times \frac{V - 1600}{1600} = \boxed{\frac{V-1600}{16}}\ \boxed{\%}$$

For $V_1 = 1832\,\text{cm}^3$ at $\text{PL} = 16\%:\ f_1 = \dfrac{1832 - 1600}{16} = 14.5\%$

For $V_2 = 2000\,\text{cm}^3$ at $m = 21\%:\ f_2 = \dfrac{2000 - 1600}{16} = 25\%$

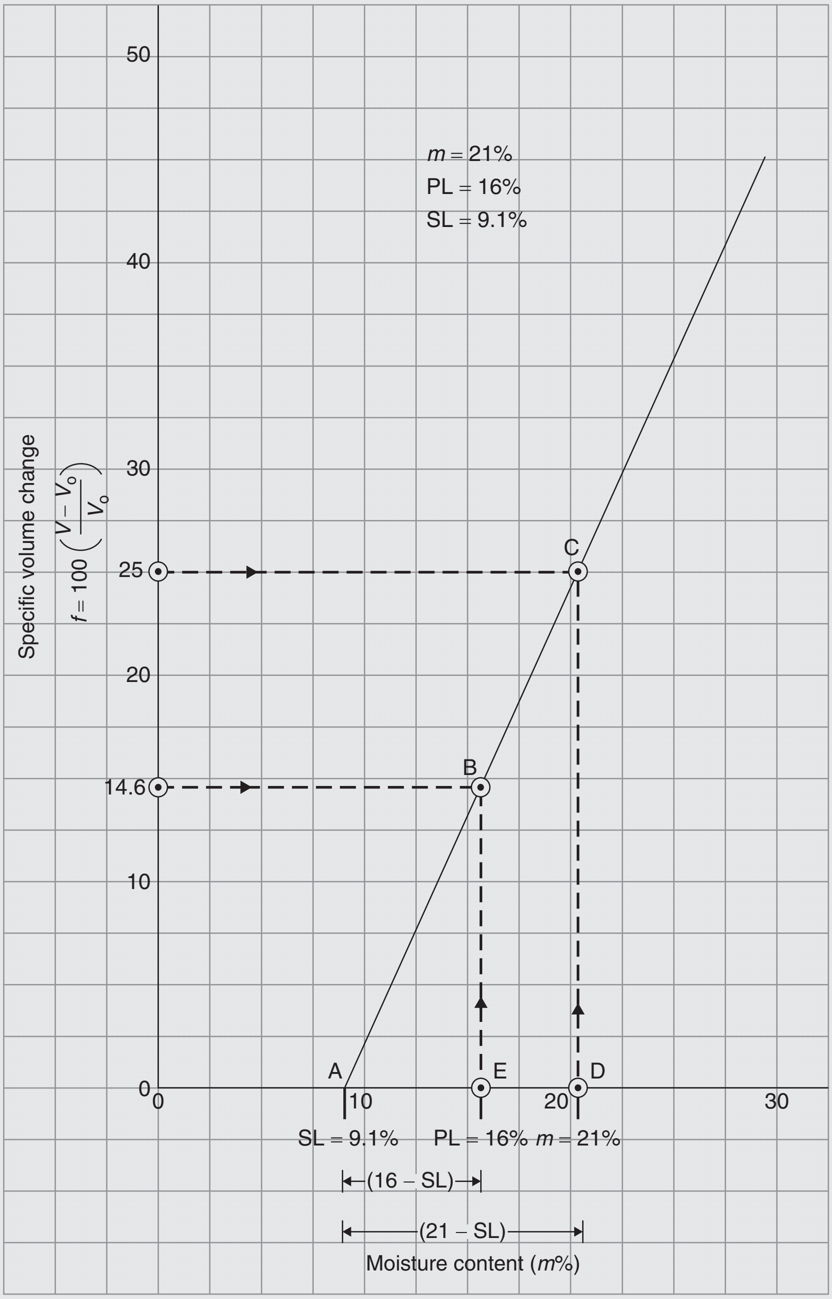
Plot these two points on Graph 2.5. The line drawn through them intersects the horizontal axis at the Shrinkage Limit, which can also be calculated from two similar triangles ABE and ACD.

$$\underset{\text{ABE}}{\frac{14.5}{16 - \text{SL}}} = \underset{\text{ACD}}{\frac{25}{21 - \text{SL}}}$$

Expressing SL:

$$14.5 \times (21 - \text{SL}) = 25 \times (16 - \text{SL})$$
$$304.5 - 14.5\,\text{SL} = 400 - 25\,\text{SL}$$
$$10.5\,\text{SL} = 95.5$$

Introduction to Soil Mechanics, First Edition. Béla Bodó and Colin Jones.



50
40
30
25
20
14.6
10
0
0
10
20
30
$m = 21\%$
PL = 16%
SL = 9.1%
Specific volume change
$f = 100\left(\frac{V - V_o}{V_o}\right)$
A
B
C
D
E
SL = 9.1%
PL = 16% $m = 21\%$
(16 – SL)
(21 – SL)
Moisture content (m%)

Graph 2.5

Therefore, the shrinkage limit is: $SL = \frac{95.5}{10.5} = 9.1\%$

b) *Voids ratios*

As the clay is fully saturated in the natural state as well as at the Shrinkage and Plastic Limits, the voids ratio at each moisture content may be calculated from:

$$(1.36):\ e = \frac{m G_s}{S_r} \quad \text{taking} \quad S_r = 1 \quad \therefore e = m G_s = 2.65m$$

For $m = 21\%$ $e_1 = 2.65 \times 0.21 = 0.56$ (56%)
For $PL = 16\%$ $e_2 = 2.65 \times 0.16 = 0.42$ (42%)
For $SL = 9.1\%$ $e_0 = 2.65 \times 0.091 = 0.24$ (24%)

Saturated unit weights

$$(1.42):\ \gamma_{sat} = \left(\frac{G_s + e}{1 + e}\right)\gamma_w = \left(\frac{2.65 + e}{1 + e}\right) \times 9.81$$

For $m = 21\%$ $\gamma_{sat1} = \left(\frac{2.65 + 0.56}{1.56}\right) \times 9.81 = 20.2\ \text{kN/m}^3$

For $PL = 16\%$ $\gamma_{sat2} = \left(\frac{2.65 + 0.42}{1.42}\right) \times 9.81 = 21.2\ \text{kN/m}^3$

For $SL = 9.1\%$ $\gamma_{sat0} = \left(\frac{2.65 + 0.24}{1.24}\right) \times 9.81 = 22.9\ \text{kN/m}^3$

Dry unit weights

For saturated soil the dry unit weight is derived from formulae (1.40) and (1.38).

$$(1.40):\ \gamma_d = \frac{\gamma}{1 + m} \quad \text{and} \quad (1.38)\ \gamma = \left(\frac{G_s + S_r e}{1 + e}\right)\gamma_w$$

But, when $S_r = 1$ $\gamma_{sat} = \left(\frac{G_s + e}{1 + e}\right)\gamma_w$

Therefore, $\boxed{\gamma_d = \frac{\gamma_{sat}}{1 + m}}$

For $m = 21\%$ $\gamma_{sat1} = 20.2\ \text{kN/m}^3 \quad \therefore \gamma_{d1} = \frac{20.2}{1.21} = 16.7\ \text{kN/m}^3$

For $PL = 16\%$ $\gamma_{sat2} = 21.2\ \text{kN/m}^3 \quad \therefore \gamma_{d2} = \frac{21.2}{1.16} = 18.3\ \text{kN/m}^3$

For $SL = 9.1\%$ $\gamma_{sat0} = 22.9\ \text{kN/m}^3 \quad \therefore \gamma_{d0} = \frac{22.9}{1.091} = 21\ \text{kN/m}^3$

Results:

a) Shrinkage Limit = 9.1%

b)

Table 2.12

	$e\%$	γ_{sat} (kN/m³)	γ_d (kN/m³)
$m=21\%$	56	20.2	16.7
PL = 16%	42	21.2	18.3
SL = 9.1%	24	22.9	21

Solution 2.2

a) Step 1: Draw the sieve-diagonals from the particle-size distributions of materials A and B.

Step 2: Plot the percentages of material C along the associated sieve-diagonal on Graph 2.6.

Step 3: Obtain the average distance of points plotted in step 2, from vertical A and draw a vertical line through it. This line represents the approximation to material C.

Step 4: From Graph 2.6:

$$a = 37.4\,\text{mm}$$
$$b = 32.6\,\text{mm}$$

The mixing ratio is given by (2.20):

$$R = \frac{b}{a} = \frac{32.6}{37.4} = 0.872$$

Step 5: Check by means of formula (2.24):

$$P = \frac{R\,P_A + P_B}{1 + R}$$

Expressing R: $P + PR = RP_A + P_B$

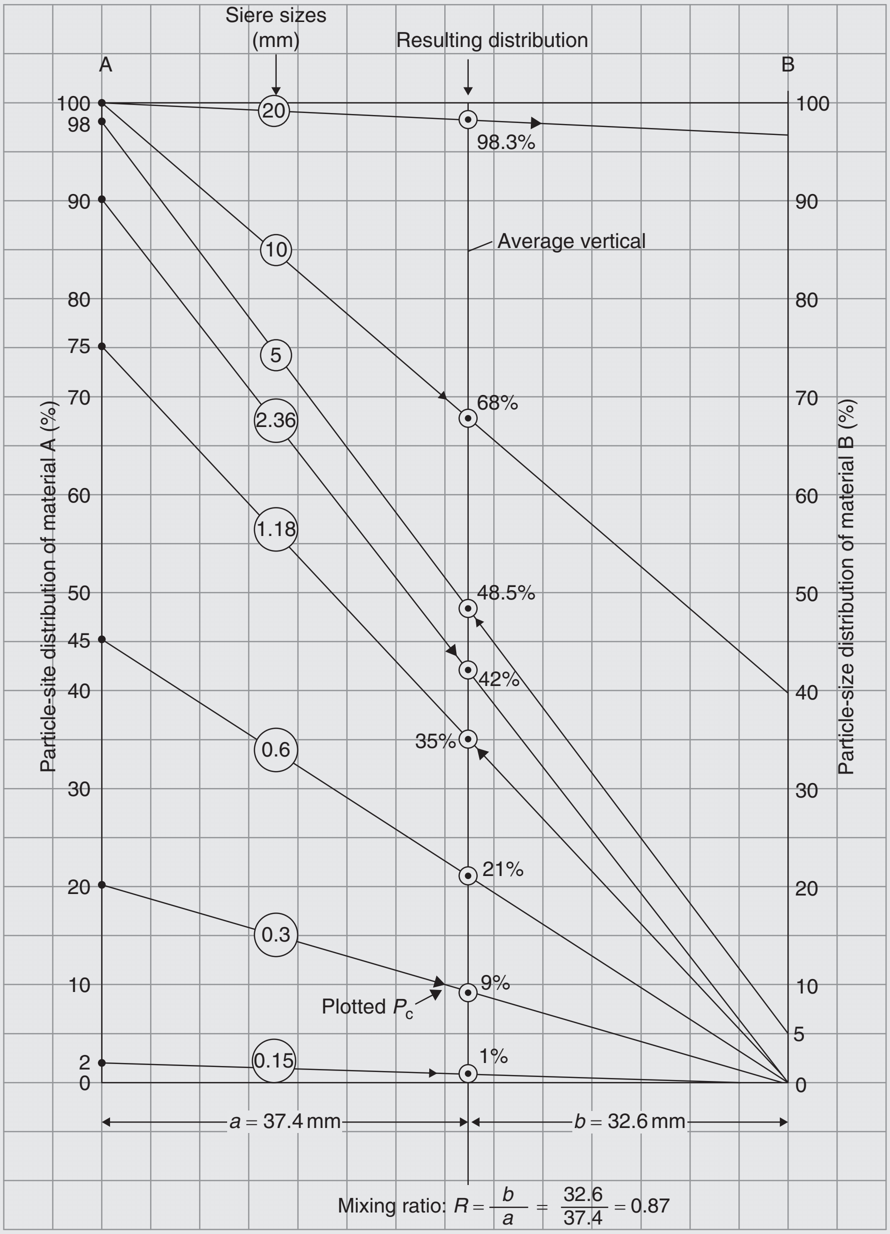
$$R(P - P_A) = P_B - P$$

$$\boxed{R = \frac{P_B - P}{P - P_A}}$$

Chose a sieve size, say the 5 mm one:
From Graph 2.6:

$P = 48.5\%$
$P_A = 98\%$
$P_B = 5\%$

$$\therefore R = \frac{5 - 48.5}{48.5 - 98} = 0.88$$


Siere sizes (mm)
Resulting distribution
A
B
100
98
90
80
75
70
60
50
45
40
30
20
10
2
0
100
90
80
70
60
50
40
30
20
10
5
0
20
10
5
2.36
1.18
0.6
0.3
0.15
98.3%
68%
48.5%
42%
35%
21%
9%
1%
Average vertical
Plotted P_c
Particle-site distribution of material A (%)
Particle-size distribution of material B (%)
$a = 37.4$ mm
$b = 32.6$ mm
Mixing ratio: $R = \frac{b}{a} = \frac{32.6}{37.4} = 0.87$

Graph 2.6

b) The distribution obtained graphically, can be checked by (2.24). All distributions are tabulated here for comparison.

Table 2.14

Sieve size (mm)	0.1	0.15	0.3	0.6	1.18	2.36	5	10	20	40
Material A: P_A %	0	2	20	45	75	90	98	100	100	100
Material B: P_B %	0	0	0	0	0	0	5	40	97	100
Resulting material $P=\frac{0.87\,P_A+P_B}{1+0.87}$ %	0	1	9	21	35	42	48	68	98	100
Distribution P_c %	0	1	10	21	34	44	48	70	98	100

Conclusion: If materials A and B are mixed so that 100 kg (say) of B is added to $0.87\times100=87$ kg of A, then the particle-size distribution of the resulting material would approximate the given distribution P_c.

Solutions to Supplementary Problems

Chapter 3

Solution 3.1

Figure 3.4 in the main text shows the sketch of the constant head permeameter.

a) The coefficient of permeability is given by:

$$(3.9):\ k=\frac{qL}{Aht}=\frac{83.30}{150\times 19.2\times 22}=0.039\,\text{cm/s}$$
$$=3.9\times 10^{-4}\,\text{m/s}$$

b) (3.2): Hydraulic gradient: $i=\frac{h}{L}=\frac{19.2}{30}=0.64$

(3.1): Discharge velocity: $\upsilon=ki=0.64\times 3.9\times 10^{-4}$
$$=2.5\times 10^{-4}\,\text{m/s}$$

c) (3.7): Seepage velocity: $\upsilon_s=\frac{ki}{n}=\frac{2.5\times 10^{-4}}{0.39}=6.4\,\text{m/s}$

d) The critical hydraulic gradient is given by:

$$(3.38):\ i_c=\frac{G_s-1}{1+e}$$
$$(1.13):\ e=\frac{n}{1-n}=\frac{0.39}{1-0.39}=0.64$$
$$\therefore\ i_c=\frac{2.66-1}{1+0.64}=1.01$$